Functional Observer-Based Event-Triggered Control for Linear Discrete-time Descriptor Systems

Jaffar Ali Lone, Shovan Bhaumik and Nutan Kumar Tomar

***Abstract*— This paper is devoted to studying the problem of functional observer-based event-triggered control (ETC) of linear discrete-time descriptor systems. As functional observers directly estimate a linear combination of states without estimating the whole state vector, we demonstrate that utilizing the functional observer's output as the control input to the plant yields enhanced control performance and requires less number of sampling events. An advantage of ETC over conventional time-triggered control lies in its ability to dynamically adjust control actions only when needed, leading to efficient resource usage. In the ETC framework, updates to the observer-based controller occur exclusively during the event-triggered sampling instants defined by some predefined event condition. In this paper, despite a substantial reduction in sampling frequency, we establish a condition for the ultimate boundedness of the closed-loop system, expressed in terms of matrix inequalities. A numerical example is presented to showcase the feasibility and effectiveness of the theoretic results.**

***Index Terms*— Descriptor systems, Estimation, Event-triggered control, Functional observers, Stability**

I. INTRODUCTION

Event-triggered control (ETC) has emerged as a revolutionary paradigm in the field of control systems, offering significant advantages over traditional periodic control methods [1], [2]. Traditional periodic control systems operate at fixed time intervals, often resulting in excessive resource consumption and suboptimal performance. In contrast, ETC dynamically adjusts the control actions only when necessary, governed by a specific event-triggered mechanism (ETM), such as changes in the system's state or external disturbances [3]. This dynamic adaptation enables more efficient resource utilization and can lead to substantial energy savings, particularly in applications where control updates are not frequently required [4]–[6]. It also offers the potential for improved system robustness and reduced wear and tear on components due to reduced actuation frequency [7].

In networked control systems, the advantages of ETC are particularly pronounced. These systems are characterized by the exchange of control information over communication networks, which can introduce delays and packet losses [8], [9]. ETC minimizes the utilization of network resources by only transmitting control updates when required, reducing network congestion and enhancing system stability [10]. This approach significantly improves the overall system's resilience to network disturbances and bottlenecks. Moreover, ETC's adaptability allows it to account for variable network conditions, ensuring that control decisions are made in a timely and efficient manner. The reduced data transmission frequency in ETC also contributes to energy savings, which is crucial in battery-powered or resource-constrained networked systems [11].

The study of descriptor systems has attracted considerable attention during the last few decades due to their strong potential applications in power systems [12], economic systems [13], smart grids [14], biological systems [15], chemical engineering [16], robotics [17] *etc*. Descriptor systems, also known as singular systems, are described by both differential and algebraic equations (DAEs). In comparison to normal state space systems (governed by ordinary differential equations (ODEs) only), descriptor systems not only preserve the structure of the physical system but also describe static constraints and impulse behaviours [16]. Despite the fact that the model reduction from DAEs to ODEs is useful for simulation studies, the reduced-order ODEs are not the exact representation of the original DAEs. Moreover, the transformed system loses the physical meaning of variables and can introduce significant numerical errors [18]. Therefore, descriptor systems should be studied in their natural form without any model transformation.

Most of the prior work on event-triggered control has operated under a common assumption: the availability of full-state information for measurement [19], [20]. In real-world applications, however, obtaining complete state information is often challenging or even impossible [21], [22]. This limitation underscores the significance of extending the event-triggered control framework to encompass output-based control with the incorporation of an observer. By incorporating observers into the control architecture, the system can effectively estimate the unmeasured states using available output measurements. This extension facilitates more widespread applicability of event-triggered control in practical scenarios, where full state measurements may be cost-prohibitive or technically infeasible. In many applications, for example, state feedback design, fault estimation, *etc.*, it is sufficient to estimate only a portion or some linear function of the states instead of estimating the whole state vector [23], [24]. Observers that estimate a linear function of state variables without estimating the vector of whole state variables are called functional (or partial state) observers. Such observers eliminate the redundancy because the full state observers may estimate even those states which are either directly measurable or are of no use [25].

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Functional observer-based ETC is well-studied in the normal state space framework [26]–[28], but in the case of descriptor systems, not a single result is published. In this paper, to the best of our knowledge, we make a first attempt to design a functional observer-based ETC for descriptor systems by considering them in their natural DAE form. It is worth mentioning here that earlier works on the ETC for descriptor systems transformed the system from DAE form to ODE form [19], [29], [30]. We establish the conditions for ensuring the ultimate boundedness of the closed-loop system, expressed in terms of matrix inequalities. Moreover, we also achieved a substantial reduction in the sampling frequency, which is demonstrated through a numerical example.

The organization of the remaining paper is as follows. In Section II, we formulate the problem of functional observer design based on ETC for descriptor systems. The functional observer design is briefly derived in Section III. The closed loop system in the ETC framework is formulated in Section IV along with its stability analysis. Section V illustrates the design approach through a numerical example. Finally, Section VI concludes the paper.

We use the following notations: 0 and I stands for appropriate dimensional zero and identity matrices, respectively. The symbol M^+ denotes the Moore-Penrose (MP) inverse of any matrix M. For a square matrix P, $P > 0$ $(P < 0)$ means that this matrix is positive definite (negative definite). The set of complex numbers is denoted by $\mathbb{C}$.

II. Problem Formulation

Fig. 1 shows the event-triggered control system considered in this paper. The dynamics of the plant is governed by the linear discrete-time descriptor system of the form:

$$Ex_{k+1} = Ax_k + Bu_k, \tag{1a}$$

$$y_k = Cx_k, \tag{1b}$$

$$z_k = Kx_k, \tag{1c}$$

where E, $A \in \mathbb{R}^{n\times n}$, $B \in \mathbb{R}^{n\times k}$, $C \in \mathbb{R}^{p\times n}$ and $K \in \mathbb{R}^{r\times n}$ are the known constant matrices. $x_k \in \mathbb{R}^n$, $u_k \in \mathbb{R}^k$ and $y_k \in \mathbb{R}^p$ are the state, the input and the output vectors respectively. $z_k \in \mathbb{R}^r$ denotes the functional vector and is defined via a linear operator K that takes x_k as input argument and returns the vector z_k. The functional vector $z_k \in \mathbb{R}^r$ contains those variables which can not be measured, and therefore, we need observers to estimate them. Any system of the form (1) is called regular if its matrix pencil $\lambda E - A$ is square and the determinant of matrix pencil does not vanish identically. Regularity ensures the existence and uniqueness of a solution of any square descriptor system. In this paper, we assume the descriptor system (1) to be regular.

In the ETC setup, the output y_k of the plant (1) is sampled only at the time instants k_s where $s = 0, 1, \ldots$ and so on. This sampled output y_{k_s} is sent to the functional observer, which is described in the interval $[k_s\ k_{s+1})$ as follows:

$$w_{k+1} = Nw_k + Ly_{k_s} + Hu_k, \tag{2a}$$

$$\hat{z}_k = w_k + My_{k_s}. \tag{2b}$$

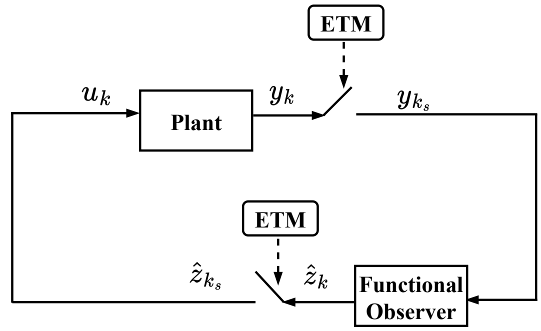

Fig. 1. Event triggered control structure

The observer given by (2) is preferred because it is easy to implement and can be initialized with arbitrary initial conditions, unlike DAE observers given in [31] due to their implicit nature and requirement of consistent initial condition. Mathematically, the meaning of the functional observer design for system (1) is to construct compatible dimension matrices N, H, L and M such that system (2) asymptotically estimates system (1), *i.e.,* $\hat{z}_k \to z_k$ as $k \to \infty$. Moreover, if $\hat{z}_j = z_j$ then $\hat{z}_k = z_k$ for all $k > j$. Any functional observer (2) reduces to a full-state observer if K is the identity matrix of the appropriate dimension.

The functional observer computes an estimate of z_k at the current sampling time. This estimated value is subsequently sampled at k_s and then transmitted to the plant as a control input. A zero-order hold mechanism is employed to keep the control input constant between the sampling time, *i.e.,*

$$u_k = \hat{z}_{k_s},\ k \in [k_s\ k_{s+1}), \tag{3}$$

where k_s is the instant when the event happens. We conclude this section by recalling some results from basic matrix theory.

Lemma 1: [32] System $\mathscr{X}\mathscr{A} = \mathscr{B}$ has a solution for $\mathscr{X}$ if and only if $\mathscr{B} = \mathscr{B}\mathscr{A}^+\mathscr{A}$, or equivalently, rank$\begin{bmatrix}\mathscr{A}\\ \mathscr{B}\end{bmatrix}$= rank$\mathscr{A}$. Moreover,

$$\mathscr{X} = \mathscr{B}\mathscr{A}^+ - \mathscr{Z}(\mathscr{I} - \mathscr{A}\mathscr{A}^+),$$

where $\mathscr{Z}$ is an arbitrary matrix of compatible dimension.

Lemma 2: [33] Let $\mathscr{X}$, $\mathscr{Y}$ and $\mathscr{W}$ be any matrices of compatible dimensions. If $\mathscr{X}$ has full row rank and/or $\mathscr{Y}$ has full column rank, then,

$$\text{rank}\begin{bmatrix}\mathscr{X} & \mathscr{W}\\ 0 & \mathscr{Y}\end{bmatrix} = \text{rank}\mathscr{X} + \text{rank}\mathscr{Y}.$$

Lemma 3: [33] Let $\mathscr{X}$ and $\mathscr{Y}$ be any two matrices of compatible dimensions. Then rank$(\mathscr{X}\mathscr{Y}) = $ rank$\mathscr{Y}$ if and only if the matrix $\begin{bmatrix}\mathscr{X}\\ I - \mathscr{Y}\mathscr{Y}^+\end{bmatrix}$ has full column rank.

III. Functional Observer Design

We assume the following two rank conditions on system (1)

$$\operatorname{rank}\begin{bmatrix} E & A \\ C & 0 \\ 0 & C \\ 0 & K \\ K & 0 \end{bmatrix} = \operatorname{rank}\begin{bmatrix} E & A \\ C & 0 \\ 0 & C \\ 0 & K \end{bmatrix}, \tag{4}$$

and, for any $\Lambda \in \mathbb{C}$, $|\Lambda| \geq 1$, Λ is finite,

$$\operatorname{rank}\begin{bmatrix} E & A \\ C & 0 \\ 0 & C \\ K & \Lambda K \end{bmatrix} = \operatorname{rank}\begin{bmatrix} E & A \\ C & 0 \\ 0 & C \\ 0 & K \end{bmatrix}. \tag{5}$$

Theorem 1: Under assumption (4) and (5), the functional observer (2) estimates z_k asymptotically if there exist a matrix T of appropriate dimension such that the following conditions hold:

$$NTE + LC = TA, \tag{6a}$$

$$TE + MC = K, \tag{6b}$$

$$H = TB, \tag{6c}$$

and matrix N is Schur.

For the detailed proof of Theorem 1, we refer to [34]; however, for the sake of completeness, we present a step-by-step procedure for the functional observer design.

Step 1: Denote $\Phi_3 = \begin{bmatrix} E & A \\ C & 0 \\ 0 & C \\ 0 & -K \end{bmatrix}$, and then compute $\Phi_1 = \begin{bmatrix} K & 0 \end{bmatrix} \Phi_3^+, \Phi_2 = I - \Phi_3\Phi_3^+$.

Step 2: Extract N_1 and N_2 from Φ_1 and Φ_2, respectively, as follows [34]

$$N_1 = \Phi_1 \begin{bmatrix} 0 & 0 & 0 & I \end{bmatrix}^T \text{ and } N_2 = \Phi_2 \begin{bmatrix} 0 & 0 & 0 & I \end{bmatrix}^T.$$

Step 3: Find matrix Z such that $N = N_1 - ZN_2$, is stable. Matrix Z can be computed by using either the pole placement technique or the linear matrix inequality approach.

Step 4: Compute the following matrices,

$$T = (\Phi_1 - Z\Phi_2)\begin{bmatrix} I & 0 & 0 & 0 \end{bmatrix}^T, \; H = TB,$$

$$M = (\Phi_1 - Z\Phi_2)\begin{bmatrix} 0 & I & 0 & 0 \end{bmatrix}^T, \; L = NM - Q,$$

$$\text{and, } Q = (\Phi_1 - Z\Phi_2)\begin{bmatrix} 0 & 0 & I & 0 \end{bmatrix}^T.$$

This completes the whole design procedure.

Now, we show that the matrix N is Schur. As given in Step 3, the matrix N is Schur if and only if the matrix pair (N_1, N_2) is detectable, *i.e.*, and for all $\Lambda \in \mathbb{C}$, $|\Lambda| \geq 1$, Λ is finite,

$$\text{matrix } \begin{bmatrix} N_1 - \Lambda I \\ N_2 \end{bmatrix} \text{ has full column rank.} \tag{7}$$

Here, we show that Eq. (7) holds if and only if (4) and (5) hold. It is clear from Lemma 1 that (4) is equivalent to the fact that

$$\begin{bmatrix} K & 0 \end{bmatrix} = \begin{bmatrix} K & 0 \end{bmatrix} \Phi_3^+ \Phi_3. \tag{8}$$

Thus, by substituting (8) in (5), it is easy to obtain that

$$\operatorname{rank}\begin{bmatrix} \begin{bmatrix} I & 0 & 0 & 0 \\ 0 & I & 0 & 0 \\ 0 & 0 & I & 0 \end{bmatrix} \\ \begin{bmatrix} K & 0 \end{bmatrix}\Phi_3^+ + \Lambda\begin{bmatrix} 0 & 0 & 0 & -I \end{bmatrix} \end{bmatrix} \Phi_3 = \operatorname{rank}\Phi_3, \tag{9}$$

which is, using Lemma 3 and definitions of Φ_1 and Φ_2, equivalent to the fact that matrix

$$\begin{bmatrix} \begin{bmatrix} I & 0 & 0 & 0 \\ 0 & I & 0 & 0 \\ 0 & 0 & I & 0 \end{bmatrix} \\ \begin{bmatrix} K & 0 \end{bmatrix}\Phi_3^+ + \Lambda\begin{bmatrix} 0 & 0 & 0 & -I \end{bmatrix} \\ I - \Phi_3\Phi_3^+ \end{bmatrix} \text{ has full column rank,}$$

$$\textit{i.e.}, \text{ matrix } \begin{bmatrix} \begin{bmatrix} I & 0 & 0 & 0 \\ 0 & I & 0 & 0 \\ 0 & 0 & I & 0 \end{bmatrix} \\ \Phi_1 + \Lambda\begin{bmatrix} 0 & 0 & 0 & -I \end{bmatrix} \\ \Phi_2 \end{bmatrix} \text{ has full column rank,}$$

$$\textit{i.e.}, \text{ matrix } \begin{bmatrix} I & 0 & 0 & 0 \\ 0 & I & 0 & 0 \\ 0 & 0 & I & 0 \\ T_1 & M_1 & Q_1 & N_1 - \Lambda I \\ T_2 & M_2 & Q_2 & N_2 \end{bmatrix} \text{ has full column rank,}$$

which is equivalent to (7) by Lemma 2.

Remark 1: It is important to note here that when $K = I$, Eqs. (4) and (5) reduce to the well-known conditions impulse observability and detectability for the system (1).

IV. Event Triggered Control

A. Closed Loop System

Let the event-triggered induced errors be defined as

$$\hat{\varepsilon}_k = y_{k_s} - y_k, \tag{10a}$$

$$\tilde{\varepsilon}_k = \hat{z}_{k_s} - \hat{z}_k. \tag{10b}$$

Using (10), the state equation of plant and the functional observer for $k \in [k_s \;\; k_{s+1})$ is reorganised as,

$$\begin{aligned} Ex_{k+1} &= Ax_k + B\tilde{\varepsilon}_k + B(w_k + My_{k_s}) \\ &= (A + BMC)x_k + Bw_k + BM\hat{\varepsilon}_k + B\tilde{\varepsilon}_k. \end{aligned} \tag{11}$$

$$\begin{aligned} w_{k+1} &= Nw_k + L\hat{\varepsilon}_k + LCx_k + H\tilde{\varepsilon}_k + H(w_k + My_{k_s}) \\ &= (N + H)w_k + (L + HM)Cx_k + (L + HM)\hat{\varepsilon}_k + H\tilde{\varepsilon}_k. \end{aligned} \tag{12}$$

Moreover, for the functional observer (2), the estimation error is given by,

$$\begin{aligned} \varepsilon_{k+1} &= w_{k+1} - TEx_{k+1} \\ &= Nw_k + Ly_{k_s} + Hu_k - T(Ax_k + Bu_k) \\ &= N\varepsilon_k + L\hat{\varepsilon}_k + (NTE + LC - TA)x_k + (H - TB)u_k \\ &= N\varepsilon_k + L\hat{\varepsilon}_k. \end{aligned} \tag{13}$$

Let $\eta_k = [x_k \ w_k \ \varepsilon_k]^T$ and $\xi_k = [\hat{\varepsilon}_k \ \tilde{\varepsilon}_k]^T$, then using (11), (12), and (13), we obtain the following closed loop system,

$$\mathbb{E}\eta_{k+1} = \mathbb{A}\eta_k + \mathbb{B}\xi_k, \tag{14}$$

where $\mathbb{E} = \begin{bmatrix} E & 0 \\ 0 & 0 \end{bmatrix}$, $\mathbb{A} = \begin{bmatrix} A+BMC & B & 0 \\ LC+HMC & N+H & 0 \\ 0 & 0 & N \end{bmatrix}$, and $\mathbb{B} = \begin{bmatrix} BM & B \\ L+HM & H \\ L & 0 \end{bmatrix}$.

Remark 2: It becomes evident that, during the sampling instances, the system described in (14) reduces into a time-triggered functional observer-based closed-loop system. Consequently, the event-triggered closed-loop system can be viewed as a time-triggered functional observer-based closed-loop system with a bounded disturbance arising from the event-triggered sampling process.

B. Event Triggering Condition

Before delving into the specific event-triggering condition, it's essential to highlight the pivotal role that such conditions play in the network control systems. Event-triggering conditions serve as the critical decision-making mechanisms that dictate when control actions should be taken in response to changes or disturbances in the system. These conditions are designed to optimize the utilization of resources, reduce the frequency of control updates, and enhance system efficiency. They are tailored to adapt to various system dynamics, ensuring that control decisions are made precisely when needed while also accommodating resource limitations, such as communication bandwidth in networked control systems. By defining a suitable event-triggering condition, control systems can achieve significant advantages, including improved performance, energy savings, and resilience to uncertainties, making them an indispensable component in modern control methodologies. In this paper, based on the closed-loop system (14), we define the following event triggering condition,

$$k_{s+1} = sup\{k > k_s | \ \xi_k^T \Omega \xi_k < \mu\}, \tag{15}$$

where $\mu > 0$ is an event triggered threshold appropriately chosen in advance and Ω is weighting matrix to be determined. The updating instants $k_s, \ s = 1, \ 2, \ \ldots,$ are determined when the event condition given by (15) is violated.

C. Stability Analysis

In this subsection, we will derive the conditions for stability of the closed-loop system (14).

Theorem 2: Consider the closed loop system (14) with the sampling instant determined by (15). If there exists a positive definite matrix $\mathbb{P}$ satisfying the following matrix inequality,

$$\Xi = \mathbb{A}^T\mathbb{P}\mathbb{A} + \mathbb{A}^T\mathbb{P}\mathbb{B}\mathbb{B}^T\mathbb{P}\mathbb{A} - \mathbb{E}^T\mathbb{P}\mathbb{E} < 0, \tag{16}$$

then the system (14) is ultimately bounded and the weighting matrix Ω in (15) is designed as $\Omega = I + \mathbb{B}^T\mathbb{P}\mathbb{B}$, where I is the identity matrix of appropriate dimension.

Proof: In proving this theorem, we make use of a fundamental result which states that for any two vectors X and Y, the inequality, $X^TY + Y^TX \le X^TX + Y^TY$ always holds [35]. Now, consider a Lyapunov function candidate $V_k = \eta_k^T\mathbb{E}^T\mathbb{P}\mathbb{E}\eta_k$, and take its increment for $k \in [k_s \ k_{s+1})$, we obtain,

$$\begin{aligned}
\Delta V &= V_{k+1} - V_k \\
&= (\mathbb{A}\eta_k + \mathbb{B}\xi)^T\mathbb{P}(\mathbb{A}\eta_k + \mathbb{B}\xi) - \eta_k^T\mathbb{E}^T\mathbb{P}\mathbb{E}\eta_k \\
&= \eta_k^T\mathbb{A}^T\mathbb{P}\mathbb{A}\eta_k + \eta_k^T\mathbb{A}^T\mathbb{P}\mathbb{B}\xi_k + \xi_k^T\mathbb{B}^T\mathbb{P}\mathbb{A}\eta_k \\
&\quad + \xi_k^T\mathbb{B}^T\mathbb{P}\mathbb{B}\xi_k - \eta_k^T\mathbb{E}^T\mathbb{P}\mathbb{E}\eta_k \\
&\le \eta_k^T\mathbb{A}^T\mathbb{P}\mathbb{A}\eta_k + \eta_k^T\mathbb{A}^T\mathbb{P}\mathbb{B}\mathbb{B}^T\mathbb{P}\mathbb{A}\eta_k + \xi_k^T\xi_k \\
&\quad + \xi_k^T\mathbb{B}^T\mathbb{P}\mathbb{B}\xi_k - \eta_k^T\mathbb{E}^T\mathbb{P}\mathbb{E}\eta_k \\
&= \eta_k^T(\mathbb{A}^T\mathbb{P}\mathbb{A} + \mathbb{A}^T\mathbb{P}\mathbb{B}\mathbb{B}^T\mathbb{P}\mathbb{A} - \mathbb{E}^T\mathbb{P}\mathbb{E})\eta_k \\
&\quad + \xi_k^T(I + \mathbb{B}^T\mathbb{P}\mathbb{B})\xi_k \\
&= \eta_k^T\Xi\eta_k + \xi_k^T\Omega\xi_k \\
&< \eta_k^T\Xi\eta_k + \mu.
\end{aligned}$$

Since $\mu > 0$ is a finite user-defined event triggered threshold, and also the matrix inequality given by (16) is satisfied. It means that ΔV decays in the interval $[k_s \ k_{s+1}), \ s = 1, \ 2, \ \ldots$ to a bounded set. Thus, with the updating instants as given by (15), the system (14) is ultimately bounded. This completes the proof. ■

Remark 3: It is important to mention here that unlike the methods given in [19], [29], [30], which require the system model transformation from DAE to ODE, we have considered the descriptor system in its natural form to preserve its authenticity and avoided any such transformation.

V. Numerical Illustration

In this section, we will demonstrate the effectiveness of our proposed functional observer-based event-triggered control through a numerical example.

Example 1: Consider the discrete-time descriptor system (1) whose coefficient matrices are given by:

$$E = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{bmatrix}, \ A = \begin{bmatrix} 0.153 & 0.045 & 0.069 \\ 0.156 & 0.252 & 0.156 \\ 0.135 & -0.171 & -0.636 \end{bmatrix},$$

$$B = \begin{bmatrix} 1 \\ 1 \\ 0.2 \end{bmatrix}, \ C = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \end{bmatrix}, \text{ and,}$$

$$K = \begin{bmatrix} 0.2693 & 0.1502 & -0.5024 \end{bmatrix}.$$

In the selection of matrix K, a crucial criterion is to ensure that the controller $u_k = Kx_k$ is configured in a manner that preserves the regularity of the closed-loop system while also guaranteeing the asymptotic convergence of its state variables. Moreover, one can easily verify that our existence conditions (4) and (5) are satisfied. Therefore, we proceed with the functional observer design and the parameter matrices are obtained as follows:

$N = [-0.2]$, $L = [5.4571 \ 3.0528]$, $H = [0.1854]$,

$T = [0.1346 \ 0.0758 \ -0.1248]$, and $M = [0.1347 \ 0.0744]$.

The value of μ is taken as 1, and the sampling time is taken as 0.1 second. The simulation results are shown in Figs. 2-6. Figs. 2-4 describe the state trajectories of the system with both time-driven and event-driven sampling. It is evident

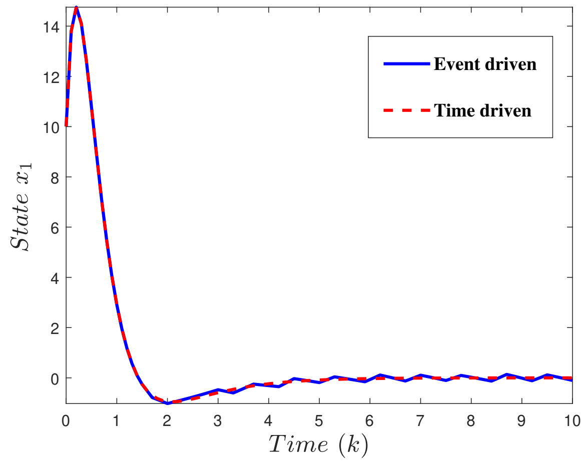

Fig. 2. State x_1 with time-driven and event-driven sampling

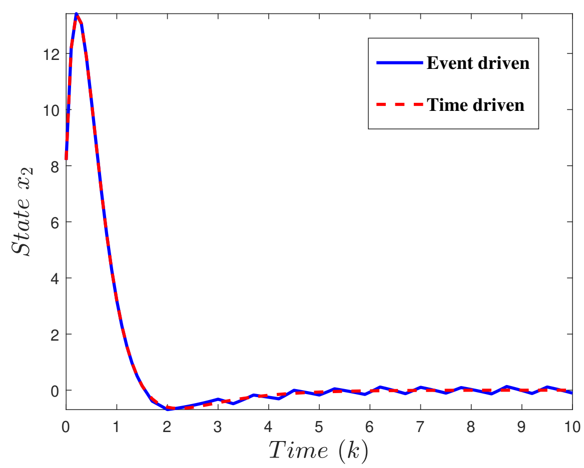

Fig. 3. State x_2 with time-driven and event-driven sampling

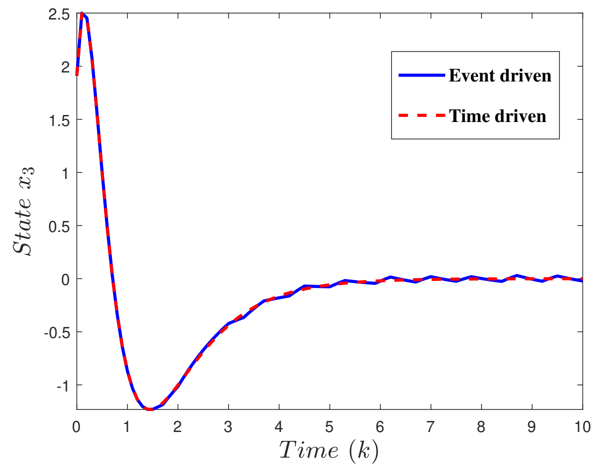

Fig. 4. State x_3 with time-driven and event-driven sampling

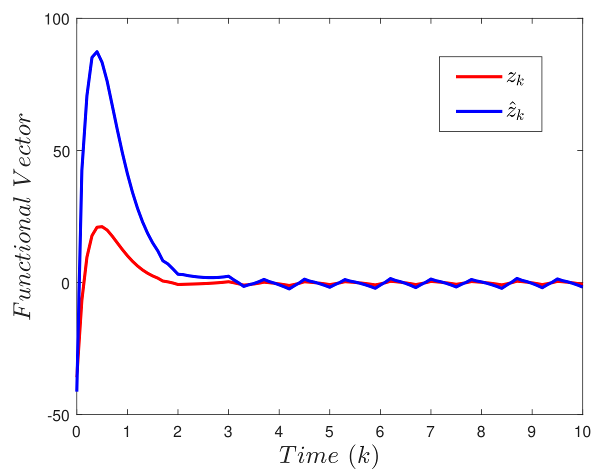

Fig. 5. True and estimated value of functional vector under ETC

from these plots that the closed-loop system is asymptotically stable. Notably, the performance achieved with event-driven sampling proved to be almost comparable to that of time-driven sampling. This observation highlights an inherent tradeoff in ETC between performance and the frequency of sampling events. While the event-driven approach offers resource-efficient control with fewer sampling instances, it maintains nearly equivalent control performance. Fig. 5 denotes the true and estimated value of the functional vector z_k under event-driven sampling. As can be seen from the plot, the estimated value quickly tracks the true value, demonstrating the accuracy of our functional observer. Moreover, it is worth mentioning here that our proposed event-triggered strategy requires only 34 sampling instants, as shown in Fig. 6, unlike the traditional time-triggered sampling, which with the sampling time of 0.1 second, would require 100 sampling instants. This illustrates the application of ETC in saving network communication bandwidth resources.

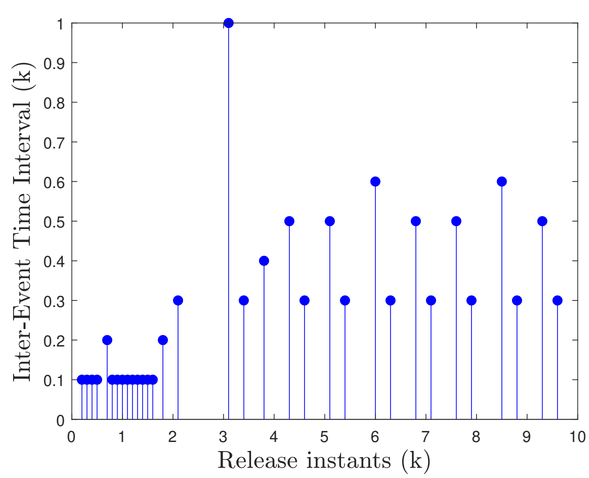

Fig. 6. Event driven transmission instants

VI. Conclusion

In this paper, we have designed a functional observer-based ETC applied to linear discrete-time descriptor systems. We highlighted a key advantage of ETC over conventional time-triggered control, emphasizing its dynamic adjustment of control actions based on system needs, resulting in more efficient resource utilization. Within the ETC framework, we have demonstrated that the observer-based controller updates occur exclusively during event-triggered sampling instants governed by predefined ETM. Notably, even with a significant reduction in the sampling frequency, we have successfully established conditions for the ultimate boundedness of the closed-loop system, expressed as matrix inequalities. Our findings are exemplified through a numerical example, showcasing the practicality and effectiveness of our theoretical contributions. In future work, we will extend the ETC methodology for nonlinear discrete-time descriptor systems with disturbances and also, to make the interevent time lower bounded we will take the Zeno phenomenon into consideration.

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